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Resonances and the Weinberg-Tomozawa 56-baryon–35-meson interaction

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Abstract. Vector meson degrees of freedom are incorporated into the Weinberg-Tomozawa (WT) meson-baryon chiral Lagrangian by using a scheme which relies on spin-flavor $SU(6)$ symmetry. The corresponding Bethe-Salpeter approximation successfully reproduces previous $SU(3)$-flavor WT results for the lowest-lying s-wave negative-parity baryon resonances, and it also provides some information on the dynamics of the heavier ones. Moreover, it also predicts the existence of an isoscalar spin-parity $\frac{3}{2}^-$ K^*N bound state (strangeness $+1$) with a mass around 1.7–1.8 GeV, unstable through K^* decay. Neglecting d-wave KN decays, this state turns out to be quite narrow ($\Gamma \leq 15$ MeV) and it might provide clear signals in reactions like $\gamma p \to \bar{K}^0 p K^+ \pi^-$ by looking at the three-body $pK^+\pi^-$ invariant mass.

PACS. 11.30.Hv Flavor symmetries – 11.10.St Bound and unstable states; Bethe-Salpeter equations – 11.30.Rd Chiral symmetries – 11.80.Gw Multichannel scattering

1 Introduction

We present results obtained from a scheme where it is assumed that the light-quark–light-quark interaction is approximately spin independent as well as $SU(3)$ independent. This corresponds to treating the six states of a light quark (u, d or s with spin up, $\uparrow$, or down, $\downarrow$) as equivalent, and leads us to the invariance group $SU(6)$. Despite the fact that the no-go Coleman-Mandula theorem [1] forbids this hybrid symmetry (mixing the compact, purely internal flavor symmetry, with the noncompact Poincare symmetry of spin angular momentum) to be exact, there exist several $SU(6)$ predictions (relative closeness of baryon octet and decuplet masses, the axial current coefficient ratio $F/D = 2/3$, the magnetic moment ratio $\mu_p/\mu_n = -3/2$) which are remarkably well satisfied in nature [2]. This suggests that $SU(6)$ could be a good approximate symmetry. Though in general the spin-flavor symmetry is not exact for excited baryons even in the large N_c limit (being N_c the number of colors)[1], in the real world ($N_c = 3$), the zeroth order spin-flavor symmetry breaking turns out to be similar in magnitude to $\mathcal{O}(N_c^{-1})$ breaking effects [5]. Spin-flavor symmetry in the meson sector is not a direct consequence of large-N_c QCD either. However, vector mesons ($K^*, \rho, \omega, \bar{K}^*, \phi$) do exist and they are known to play a relevant role in hadronic

physics. Inescapably, they will couple to baryons and will presumably influence the properties of the baryonic resonances. Lacking better theoretically founded models to include vector mesons, we regard the spin-flavor symmetric scenario as reasonable first step. The large-N_c consequences of this scheme have been pursued in [6].

We will consider the s-wave interaction between the $SU(6)$ lowest-lying meson multiplet (**35**) and the lowest-lying baryons (**56**-plet) at low energies. The meson multiplet contains the octet of pseudoscalar ($K, \pi, \eta, \bar{K}$) and the nonet of vector ($K^*, \rho, \omega, \bar{K}^*, \phi$) mesons, while the baryon one is constructed from the (N, Σ, Λ, Ξ) octet of spin-1/2 baryons and the ($\Delta, \Sigma^*, \Xi^*, \Omega$) decuplet of spin-3/2 baryons. Assuming that the s-wave effective meson-baryon Hamiltonian is $SU(6)$ invariant, and since the $SU(6)$ decomposition of the product of the **35** (meson) and **56** (baryon) representations yields

$$\mathbf{35} \otimes \mathbf{56} = \mathbf{56} \oplus \mathbf{70} \oplus \mathbf{700} \oplus \mathbf{1134}, \qquad (1)$$

there are only four, Wigner-Eckart irreducible matrix elements (WEIMEs), free functions of the meson-baryon Mandelstam variable s. Similar ideas were already explored in the late sixties, within the effective-range approximation [7]. In this work, based on the findings of ref. [8], two major improvements have been introduced: i) The use of the underlying Chiral Symmetry (CS), which would allow to determine the value of the $SU(6)$ irreducible matrix elements from the Weinberg-Tomozawa (WT) interaction (leading term of the chiral Lagrangian involving Goldstone bosons and the octet of spin-1/2

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[1] In the large-N_c limit [3,4], there exists an exact spin-flavor symmetry for ground-state baryons.

baryons), ii) the use of Bethe-Salpeter Equations (BSE) [9, 10], in coupled channels and with an appropriated Renormalization Scheme (RS) [11,12], to determine the scattering amplitudes, going thus beyond the effective-range approximation.

2 $SU(6)$ extension of the meson-baryon WT interaction

We will work with well-defined total isospin (I), angular momentum (J) and hypercharge (Y) meson-baryon states constructed out of the $SU(6)$ **35** (mesons) and **56** (baryon) multiplets. By imposing that the effective s-wave meson-baryon *potential* (V) is a $SU(6)$ invariant operator, we find [8]

$$\langle \mathcal{M}'\mathcal{B}'; JIY|V|\mathcal{MB}; JIY \rangle = \sum_\phi V_\phi(s) \mathcal{P}^{\phi,JIY}_{\mathcal{MB},\mathcal{M}'\mathcal{B}'}, \quad (2)$$

where $\mathcal{M} \equiv [(\mu_M)_{2J_M+1}, I_M, Y_M]$ stands for meson states and similarly $\mathcal{B}$ for baryon ones and the labels μ and ϕ denote $SU(3)$ and $SU(6)$ representations, respectively. In the above equation ϕ runs over the **56**, **70**, **700** and **1134** irreducible representations (irreps), as inferred from eq. (1). The projectors are given in terms of the $SU(3)$ isoscalar [13], and the $SU(6)$-multiplet coupling [14,15] factors,

$$\mathcal{P}^{\phi,JIY}_{\mathcal{MB},\mathcal{M}'\mathcal{B}'} = \sum_{\mu,\alpha} \begin{pmatrix} \mathbf{35} & \mathbf{56} \\ \mu_M J_M & \mu_B J_B \end{pmatrix} \begin{vmatrix} \phi \\ \mu J\alpha \end{pmatrix}$$

$$\times \begin{pmatrix} \mu_M & \mu_B \\ I_M Y_M & I_B Y_B \end{pmatrix} \begin{vmatrix} \mu \\ IY \end{pmatrix} \begin{pmatrix} \mu'_{M'} & \mu'_{B'} \\ I'_{M'} Y'_{M'} & I'_{B'} Y'_{B'} \end{pmatrix} \begin{vmatrix} \mu \\ IY \end{pmatrix}$$

$$\times \begin{pmatrix} \mathbf{35} & \mathbf{56} \\ \mu'_{M'} J'_{M'} & \mu'_{B'} J'_{B'} \end{pmatrix} \begin{vmatrix} \phi \\ \mu J\alpha \end{pmatrix}, \quad (3)$$

where α accounts for the multiplicity of each of the μ_{2J+1} $SU(3)$ multiplets of spin J (for $L = 0$, J is given by the total spin of the meson-baryon system) entering in the representation ϕ. The $SU(6)$ WEIMEs, $V_\phi(s)$, might be constrained by demanding that the above interaction, when restricted to the Goldstone pseudoscalar meson and the lowest $J^P = \frac{1}{2}^+$ baryon octet space, will reduce to that deduced from $SU(3)$ chiral symmetry. At leading order in the chiral expansion, this latter one is obtained from the WT Lagrangian, which besides hadron masses only depends on the $f \simeq 93\,\mathrm{MeV}$, the pion weak decay constant, and it can be exactly recovered from eq. (2) by setting the WEIMEs as follows [8]:

$$V_\phi(s) = \bar{\lambda}_\phi \frac{\sqrt{s} - M}{2 f^2}, \quad (4)$$

with $\bar{\lambda}_{\mathbf{56}} = -12$, $\bar{\lambda}_{\mathbf{70}} = -18$, $\bar{\lambda}_{\mathbf{700}} = 6$ and $\bar{\lambda}_{\mathbf{1134}} = -2$ and M the common octet and decuplet baryon mass[2]. This

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[2] The $SU(6)$ extension thus obtained (eqs. (2) and (4)) also leads to the *potentials* used in refs. [16,17] to study the $(\mathbf{10_4})$baryon-$(\mathbf{8_1})$meson sector.

is not a trivial fact and it is intimately linked to the group structure of the WT term. Indeed, the underlying reason for this is CS, since the WT Lagrangian is not just $SU(3)$ symmetric but also chiral $(SU_L(3) \otimes SU_R(3))$ invariant [8].

3 Meson-baryon scattering matrix

We solve the coupled-channel BSE with an interaction kernel determined by eqs. (2) and (4). In a given JIY sector, the solution for the coupled-channel s-wave scattering amplitude, $T^J_{IY}(\sqrt{s})$ (normalized as the t-matrix defined in eq. (33) of [18]), in the *on-shell* scheme [9,10,19,11,20] reads,

$$T^J_{IY}(\sqrt{s}) = \frac{1}{1 - V^J_{IY}(\sqrt{s}) J^J_{IY}(\sqrt{s})} V^J_{IY}(\sqrt{s}) \quad (5)$$

with $V^J_{IY}(\sqrt{s}) = \langle \mathcal{M}'\mathcal{B}'; JIY|V|\mathcal{MB}; JIY \rangle$, and $J^J_{IY}(\sqrt{s})$ a diagonal matrix of loop functions [18,21]. Those are logarithmically divergent and hence to make them finite an ultraviolet (UV) cutoff or a subtraction point μ^{JIY}_i, such that

$$\left[J^J_{IY}(\sqrt{s} = \mu^{JIY}_i) \right]_{ii} = 0 \quad (6)$$

with the index i running in the coupled-channel space, is needed.

By setting $\mu^{JIY}_i = \sqrt{M_i^2 + m_i^2}$, with M_i and m_i the masses of the baryon and meson entering in the channel i of the sector JIY, we recover previous results, deduced from the WT $SU(3)$ chiral Lagrangian [11,12, 18–21], and make new predictions. For instance, in the $I = 0, S = -1$ $(Y = 0)$ sector, looking for poles in the second Riemann sheet:

– For $J^P = \frac{1}{2}^-$, we obtain the $\Lambda(1390), \Lambda(1405), \Lambda(1670)$ resonances, but we also find strength around
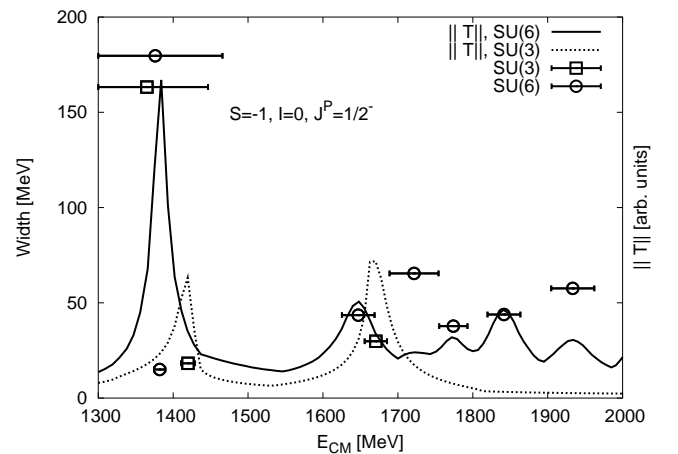

Fig. 1. Spin-parity $J^P = \frac{1}{2}^-$ resonance properties in the $I = 0, S = -1$ $(Y = 0)$ sector. Solid and dotted lines stand for the $SU(6)$ and $SU(3)$ T-matrix norms, respectively. The norm is calculated as $\|T\| = \mathrm{Max}(\sum_{j=1}^n |T_{ij}|, i = 1, \ldots, n)$ where i, j run over the number of coupled channels. Finally, the points with error bars are defined from the masses and widths of the found resonances as $(M_R \pm \Gamma_R/2, \Gamma_R)$.

1800 MeV, which might correspond to the three star $\Lambda(1800)$, which has a sizeable $N\bar{K}^*$ coupling (see fig. 1).

– For $J^P = \frac{3}{2}^-$, we get signals for some *d*-wave resonances (not accessible with the $SU(3)$ WT chiral Lagrangian): the four star $\Lambda(1520)$ and $\Lambda(1690)$ states with large couplings to the $\Sigma^*\pi$ channel, and the $\Lambda(2325)$ resonance which couples to $\Lambda\omega$ channel.

Similar results are found for the other I, Y sectors. $SU(6)$ symmetry breaking effects such that the use of a common weak decay constant (f) for all channels lead to changes of the order of 30% in resonance widths and excitation energies. These uncertainties are similar to those stemming from the use of different UV cutoffs or subtraction points to renormalize the amplitudes.

4 Exotic states

The $SU(6)$ meson-baryon interaction constructed in sect. 2 turns out to be attractive in the **1134**-irrep space ($\bar{\lambda}_{\mathbf{1134}} = -2$), which might lead to the existence of some exotic states. For instance, in the $Y = -3, I = J = 1/2$ sector, we have $|\bar{K}^*\Omega\rangle = |\mathbf{1134}; \mathbf{35_2}\rangle$ or for $Y = +2, I = 0, J = 3/2$ $|K^*N\rangle = -|\mathbf{1134}; \mathbf{10_4^*}\rangle$, hence one finds attractive $\bar{K}^*\Omega$ or K^*N interactions with these exotic quantum numbers. Whether these interactions are strong enough or not to bind the meson-baryon system will depend on the RS employed to make finite the BSE and on the nature of the further *s*- and *d*-wave contributions to the interaction matrix.

Neglecting any further correction to the *potential*, we present here results for the K^*N system in the $I = 0, J = 3/2$ sector (see ref. [8] for some more details). We find a pole in the first Riemann sheet corresponding to a K^*N bound state which we call Θ^{*+}. This state is unstable since the K^* decays into $K\pi$. To estimate the Θ^{*+} width, we model the Θ^*NK^* coupling as

$$\mathcal{L}_{\Theta^*NK^*} = -\frac{g}{\sqrt{2}}\overline{\Theta}^\mu \left(K_\mu^{*0}p - K_\mu^{*+}n \right) + \text{h.c.}, \quad (7)$$

where Θ^μ is a Rarita-Schwinger field, p and n the nucleon fields, K_μ^{*0} and K_μ^{*+} the Proca fields which annihilate and create neutral and charged K^* and $\bar{K}^*$ mesons. The subsequent K^* decay is described following ref. [22]. The coupling g is determined by the residue at the pole of $T_{02}^{\frac{3}{2}}$ (*i.e.*, $T_{02}^{\frac{3}{2}} \approx g^2 \times 2M_{\Theta^*}/(s - M_{\Theta^*}^2)$).

Resonance mass, residue and width depend on the RS employed. We have used an UV cutoff (Λ) to evaluate the loop function $J(\sqrt{s})$, which is equivalent to choose a scale $\bar{\mu}$ such that $J(\sqrt{s} = \bar{\mu}) = 0$. Results are shown in fig. 2. For $\bar{\mu}$ ranging from 0.05 GeV ($\Lambda \approx 1.08$ GeV) to 1.7 GeV ($\Lambda \approx 0.46$ GeV) the resonance mass (width) varies from 1.688 GeV (0.3 MeV), close to the ($M_N + m_\pi + m_K$) threshold, to 1.831 GeV (9 MeV, but the width does not grow monotonously, see figure), close to the ($M_N + m_{K^*}$) threshold. Other mechanisms for K^*N scattering (*d*-wave KN,
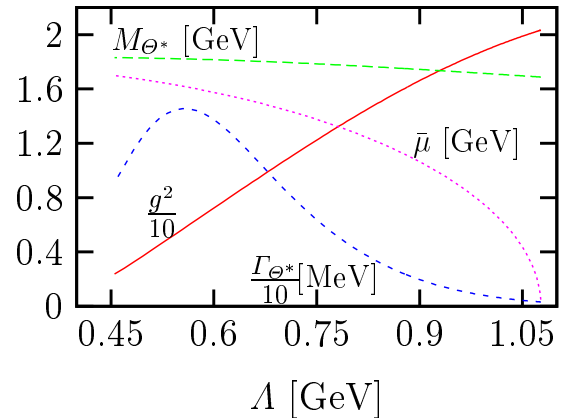

Fig. 2. Resonance Θ^{*+} properties as a function of the UV cutoff Λ or the subtraction scale $\bar{\mu}$.

K^*N contributions, *u*-channel pole graph, single pion exchange between K^* and N, sequential exchange of two pions with an intermediate K meson, corresponding to a box graph $K^*N \to KN \to K^*N$, ...) might quantitatively modify these results. However, we do not expect such corrections to be large enough to affect the existence of the Θ^* pentaquark [8]. Possible production and identification mechanisms for this resonance could be found in reactions like $\gamma p \to \bar{K}^0 pK^+\pi^-$ by measuring the three-body $pK^+\pi^-$ invariant mass.

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